STA 314 Tutorial 4

David Veitch

University of Toronto

daveveitch.github.io

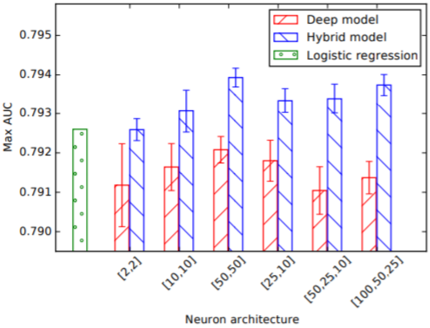
October 4, 2019

Agenda

1. Cool Paper
2. Prediction v Inference
3. Kahoot
4. ISLR 3.7 Exercise 5
5. ISLR 3.7 Exercise 7

Cool Paper

Abstract We describe the Customer LifeTime Value (CLTV) prediction system deployed at ASOS.com, a global online fashion retailer. CLTV prediction is an important problem in e-commerce where an accurate estimate of future value allows retailers to effectively allocate marketing spend, identify and nurture high value customers and mitigate exposure to losses.
https://arxiv.org/pdf/1703.02596.pdf



Performance With NN-Generated + Regular Features for Logistic Regression

Prediction v Inference

Prediction	Inference

Which is more important?

Prediction v Inference

Prediction	Inference
What is the weather tomorrow?	What is the weather in 100 years?

Which question is more important?

Kahoot

ISLR 3.7 Exercise 5

Consider the fitted values that result from performing linear regression without an intercept. In this setting the $i^t h$ fitted value takes the form

$$\hat{y}_i = x_i \hat{\beta}$$

where

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i'=1}^{n} x_{i'}^2}$$

Show that we can write

$$\hat{y}_i = \sum_{i'=1}^{n} a_{i'} y_{i'}$$

And show what does $a_{i'}$ equal

ISLR 3.7 Exercise 5

We are given that for a no intercept model, i^{th} fitted value

$$\hat{y}_i = x_i \hat{\beta}$$

where

$$\hat{\beta} = \frac{\left(\sum_{i=1}^{n} x_i y_i\right)}{\left(\sum_{i'=1}^{n} x_{i'}^2\right)}$$

we can re-write this as

$$\hat{\beta} = \frac{\left(\sum_{i'=1}^{n} x_{i'} y_{i'}\right)}{\left(\sum_{j=1}^{n} x_j^2\right)}$$

Then

$$\hat{y}_i = x_i \frac{\left(\sum_{i'=1}^{n} x_{i'} y_{i'}\right)}{\left(\sum_{j=1}^{n} x_j^2\right)} = \sum_{i'=1}^{n} \left(\frac{x_i x_{i'}}{\left(\sum_{j=1}^{n} x_j^2\right)} \right) y_{i'}$$

$$\hat{y}_i = \sum_{i'=1}^{n} a_{i'} y_{i'}$$

$$a_{i'} = \frac{x_i x_{i'}}{\left(\sum_{j=1}^{n} x_j^2\right)}$$

Note: *We interpret this result by saying that the fitted values from linear regression are linear combinations of the response values.*
Question: If for a given $\hat{y}_i$, we have x_i relatively very large what happens?

ISLR 3.7 Exercise 7

It is claimed in the text that in the case of simple linear regression of Y onto X, the R^2 statistic (3.17) is equal to the square of the correlation between X and Y (3.18). Prove that this is the case. For simplicity, you may assume that $\bar{x} = \bar{y} = 0$.

ISLR 3.7 Exercise 7

First we know that since $\bar{y} = 0$ that

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i \left((y_i - \bar{y})^2\right)} = 1 - \frac{\sum_i \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right)^2}{\sum_i y_i^2}$$

Also we know that since $\bar{x} = \bar{y} = 0$ then

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 0$$

This implies that

$$R^2 = 1 - \frac{\sum_i \left(y_i - \hat{\beta}_1 x_i\right)^2}{\sum_i y_i^2}$$

Also,

$$\left(y_i - \hat{\beta}_1 x_i\right)^2 = y_i^2 - 2x_i y_i \hat{\beta}_1 - \hat{\beta}_1^2 x_i^2$$

Taking sum over both sides gets us

$$\sum_i \left(y_i - \hat{\beta}_1 x_i\right)^2 = \sum_i y_i^2 - 2\hat{\beta}_1 \sum_i x_i y_i + \hat{\beta}_1^2 \sum_i x_i^2$$

We know since $\bar{x} = \bar{y} = 0$ that

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - x)^2} = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$$

ISLR 3.7 Exercise 7

Then substituting this into the previous equation

$$\sum_i \left(y_i - \hat{\beta}_1 x_i\right)^2 = \sum_i y_i^2 - 2\left(\frac{\sum_i x_i y_i}{\sum_i x_i^2}\right)\sum_i x_i y_i + \left(\frac{\sum_i x_i y_i}{\sum_i x_i^2}\right)^2 \sum_i x_i^2$$

$$= \sum_i y_i^2 - \frac{\left(\sum_i x_i y_i\right)^2}{\sum_i x_i^2}$$

Bringing this back to the equation for R^2 we get

$$R^2 = 1 - \frac{\sum_i \left(y_i - \hat{\beta}_1 x_i\right)^2}{\sum_i y_i^2} = \frac{\sum_i y_i^2 - \sum_i y_i^2 + \frac{\left(\sum_i x_i y_i\right)^2}{\sum_i x_i^2}}{\sum_i y_i^2} = \frac{\left(\sum_i x_i y_i\right)^2}{\sum_i x_i^2 \sum_i y_i^2}$$

ISLR 3.7 Exercise 7

Finally! We compare this last equation to the correlation equation.

From previous page

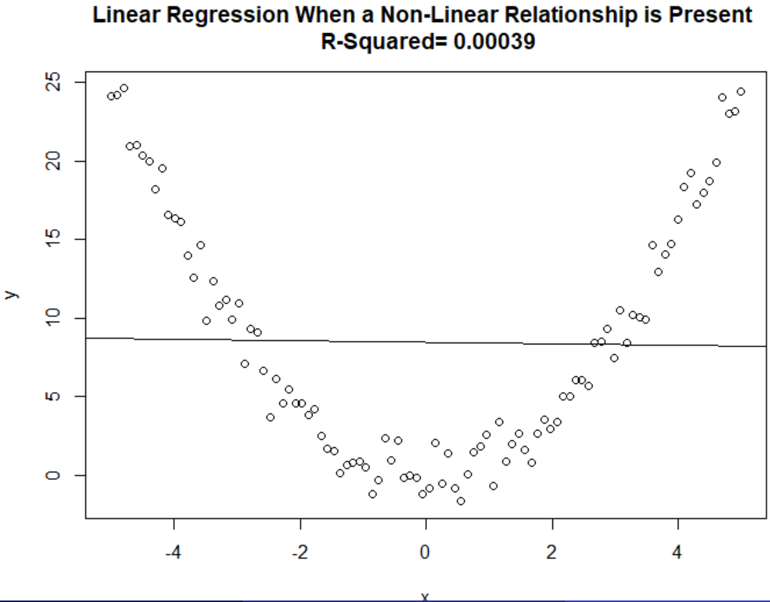
$$R^2 = \frac{\left(\sum_i x_i y_i\right)^2}{\sum_i x_i^2 \sum_i y_i^2}$$

The formula for correlation (and set $\bar{x} = \bar{y} = 0$)

$$Cor\left(X, Y\right) = \frac{\sum_i \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum_i \left(x_i - x\right)^2}\sqrt{\sum_i \left(y_i - y\right)^2}} = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2 \sum_i y_i^2}}$$

$$Cor\left(X, Y\right)^2 = \left(\frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2 \sum_i y_i^2}}\right)^2 = \frac{\left(\sum_i x_i y_i\right)^2}{\sum_i x_i^2 \sum_i y_i^2} = R^2$$

Beware R^2


Linear Regression When a Non-Linear Relationship is Present
R-Squared= 0.00039
y
0
5
10
15
20
25
-4
-2
0
2
4
x