STA 255 Tutorial 4

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Agenda

1. Midterm Info
2. Kahoot
3. Discrete Probability Example
4. Previous Midterm Question 5

Midterm Info (from Quercus)

The midterm will be on **Tuesday, July 23rd, 2019, starting at 14:00 sharp. The location of the midterm is in EX100**. The coverage of the exam is **Lecture 1 - Lecture 6 (inclusive)**. You have 120 minutes to complete five questions. Maximum attainable points is 100. The exam is closed-book. You may bring a non-programmable and non-communicable calculator. An aid sheet will be provided by the instructor and is available for preview on Quercus. Note that you **may NOT bring your printed copy of the aid sheet to the exam**. Solutions to the in-tutorial assignments and suggested textbook problems have been posted on Quercus. A sample exam by Dr. Bethany White is posted, but there is no solutions available. There is no guarantee that the actual exam will be similar to the sample exam. Please work on as many problems as you can and stop by the office hours if you have any questions. Additional office hours have been posted on Quercus in a previous announcement. To be fair to everyone in the class, I will not answer any questions regarding the details of the midterm either in person or through email. **If you have any questions on the materials, please bring your questions to office hours**. As stated in the syllabus, neither the TAs nor I will answer questions on course materials through email.

Kahoot

Suppose that 20% of all individuals have an adverse reaction to a particular drug. A medical researcher will administer the drug to one individual after another until the first adverse reaction occurs. Define an appropriate random variable and use its distribution to answer the following questions:

a) What is the probability that when the experiment terminates, four individuals have not had adverse reactions?
b) What is the probability that the drug is administered to exactly five individuals?
d) How many individuals would you expect to not have an adverse reaction, and to how many individuals would you expect the drug to be given?

a) (Lecture 4 Slide 34) A **geometric experiment** is a sequence of bernoulli trials (n not fixed) that continues until the first success is observed.

- indepdent trials
- two outcomes
- constant p across trials

X=total number of trials up to and including the first success. If X a geometric random variable it has a probability mass function:

$$\mathrm{P}(X = x) = (1 - p)^{x-1}(p)$$

Experiment terminates and four have not had adverse reactions $\Rightarrow$ fifth individual had an adverse reaction.

$$P(X = 5) = (1 - .2)^{5-1}(.2) = (.8)^4(.2) = 0.08192$$

b) Probability drug administered to exactly five individuals.

	Person Number						
	1	2	3	4	5	6	7
Experiment 1	X	X	✓				
Experiment 2	X	X	X	X	X	X	✓
Experiment 3	X	X	X	X	✓		

Same as four individuals not having an adverse reaction and the fifth having an adverse reaction. As before $P(X = 5) = 0.08192$.

d) How many individuals would you expect to not have an adverse reaction, and to how many individuals would you expect the drug to be given?

(Lecture 3 Slide 34) Expectation of a function of a discrete random variable X which takes a set of possible values D:

$$E[h(X)] = \sum_{x \in D} h(x)p(x)$$

$$\begin{aligned} E[\#\text{ Not Have Adverse}] &= 0(0.2) + 1(0.8)(0.2) + 2(0.8)^2(0.2) + \cdots \\ &= (0.2)(0.8)\sum_{x=1}^{\infty} x(0.8)^{x-1} \end{aligned}$$

$$\sum_{x=1}^{\infty} x(0.8)^{x-1} = 1 + 2(0.8) + 3(0.8)^2 + \cdots$$

$$(0.8)\sum_{x=1}^{\infty} x(0.8)^{x-1} = 0 + 0.8 + 2(0.8)^2 + 3(0.8)^3 + \cdots$$

$$\sum_{x=1}^{\infty} x(0.8)^{x-1} - (0.8)\sum_{x=1}^{\infty} x(0.8)^{x-1} = 1 + (0.8) + (0.8)^2 + \cdots$$

$$(0.2)\sum_{x=1}^{\infty} x(0.8)^{x-1} = \frac{1}{1-.8} = \frac{1}{.2} = 5$$

$$E[\# \text{ Not Have Adverse}] = 0(0.2) + 1(0.8)(0.2) + 2(0.8)^2(0.2) + \cdots$$

$$= (0.2)(0.8)\sum_{x=1}^{\infty} x(0.8)^{x-1} = (0.8)5 = 4$$

$$
\begin{aligned}
E[\#\text{ Drug Given}] &= 1(0.2) + 2(0.8)(0.2) + 3(0.8)^2(0.2) + \cdots \\
&= (0.2)\sum_{x=1}^{\infty} x(0.8)^{x-1} \\
&= 5
\end{aligned}
$$

Previous Midterm Question 5

5. The moment generating function of a Binomial random variable is:

$$M_X(t) = (pe^t + 1 - p)^n$$

Use this mgf to derive the variance of a Binomial random variable with parameters n and p.

Previous Midterm Question 5

Recall from Lecture 3 Slide 44 that

$$Var(X) = E(X^2) - E(X)^2$$

Also recall from Lecture 3 Slide 53 that

$$E(X^r) = \frac{\partial^r}{\partial x^r} M_X(0)$$

Previous Midterm Question 5

$$E(X^r) = \frac{\partial^r}{\partial t^r} M_X(0)$$

Using this formula we can find:

$$\begin{aligned} E[X] &= np \\ E[X^2] &= n(n-1)p^2 + np \\ Var[X] &= np(1-p) \end{aligned}$$