# STA 255 Tutorial 6

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# Agenda

1. Kahoot
2. Change of Variables (pg 221 Devore & Berk)
3. Chg of Variables Example
4. Joint Probability Example (Exercise 5.1.1 Devore & Berk)

# Kahoot

Where $y = g(x)$ and $x = h(y) = g^{-1}(y)$

$$F_Y(y) = P(Y \leq y) = P[g(X) \leq y] = P[X \leq h(y)] = F_x[h(y)]$$
$$F_Y(y) = F_x[h(y)]$$

Now differentiate both sides!

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$$F_Y(y) = F_x[h(y)]$$

Now differentiate both sides!

$$\begin{aligned}
\frac{d}{dy} F_Y(y) &= \frac{d}{dy} F_x[h(y)] \\
f_Y(y) &= \big[\frac{d}{dx} F_X(x)\big] \big| \frac{d}{dy} h(y) \big| \\
f_Y(y) &= f_X(x) |h'(y)| \\
f_Y(y) &= f_X[h(y)] |h'(y)|
\end{aligned}$$

# Chg of Variables Example

Let X be a random variable, where $0 \leq x \leq 1$ with pdf $f_X(x) = 2x$. Let $Y = 3x + 3$. What is the pdf of $Y$? Draw it and compare it to $X$'s pdf.

# Chg of Variables Example

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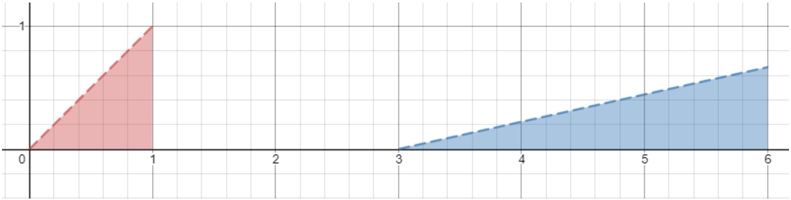

A service station has both self-service and full service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

| $p(x, y)$ | | $y = 0$ | $y = 1$ | $y = 2$ |
|---|---|---|---|---|
| $x$ | 0 | .10 | .04 | .02 |
| | 1 | .08 | .20 | .06 |
| | 2 | .06 | .14 | .30 |

1. Find $P(X = 1 \cap Y = 1), P(X \leq 1 \cap Y \leq 1), P(X \neq 0 \cap Y \neq 0)$
2. Compute the marginal pmf of $X$ and $Y$