STA 255 Tutorial 8

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Agenda

1. Kahoot
2. Estimators (Exercise 7.11 Devore & Berk)
3. Estimators (Exercise 7.23 Devore & Berk)

Kahoot

Of n_1 randomly selected male smokers, X_1 smoked filter cigarettes, whereas of n_2 randomly selected female smokers, X_2 smoked filter cigarettes. Let p_1 and p_2 denote the probabilities that a randomly selected male and female, respectively, smoke filter cigarettes.

1. Show that $X_1/n_1 - X_2/n_2$ is an unbiased estimator for $p_1 - p_2$
2. What is the standard error of the estimator in part 1
3. How would you use the observed values x_1 and x_2 to estimate the standard error of your estimator
4. If $n_1 = n_2 = 200, x_1 = 127, x_2 = 176$ use the estimator of part 1 to obtain an estimate of $p_1 - p_2$
5. Use the result of part 3 and part 4 to estimate the standard error of the estimator

1.

$$
\begin{aligned}
E[X_1/n_1 - X_2/n_2] &= E[X_1/n_1] - E[X_2/n_2] \\
&= \frac{1}{n_1}E[X_1] - \frac{1}{n_2}E[X_2] \\
&= p_1 - p_2
\end{aligned}
$$

2. Using the fact that X_1 and X_2 are independent, and the formula for the variance of a binomial distribution, we get:

$$
\begin{aligned}
Var(X_1/n_1 - X_2/n_2) &= Var(X_1/n_1) + Var(X_2/n_2) \\
&= \frac{1}{n_1^2}Var(X_1) + \frac{1}{n_2^2}Var(X_2) \\
&= \frac{n_1 p_1(1-p_1)}{n_1^2} + \frac{n_2 p_2(1-p_2)}{n_2^2} = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}
\end{aligned}
$$

3. Set $\hat{p}_1 = x_1/n_1$, $\hat{p}_2 = x_2/n_2$
4. $\hat{p}_1 - \hat{p}_2 =$ -.245
5. Plugging in $\hat{p}_1$ and $\hat{p}_2$ we get the standard error to be 0.041

Let X denote the proportion of alloted time that a randomly selected student spends working on a certain aptitutde test. Suppose the pdf of X, where $-1 < \theta$, is:

$$f(x;\theta) = \begin{cases} (\theta+1)x^{\theta} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

A random sample of 10 students yields data: .92, .79, .90, .65, .86, .47, .73, .97, .94, .77.

1. Use the method of moments to obtain an estimator of θ, and then compute the estimate for this data.
2. Obtain the maximum likelihood estimator of θ, and then compute the estimate for the given data.

1.

$$E[X] = \int_0^1 x(\theta+1)x^\theta dx = 1 - \frac{1}{\theta+2}$$
$$\Rightarrow \hat{\theta} = \frac{1}{1-\bar{X}} - 2 = \frac{1}{1-.8} - 2 = 3$$

2.

$$f(x_1,\ldots,x_n;\theta) = (\theta+1)^n(\prod_{i=1}^n x_i)^\theta$$
$$\ln\left(f(x_1,\ldots,x_n;\theta)\right) = n\ln(\theta+1) + \theta\sum_{i=1}^n \ln(x_i)$$
$$\frac{\partial}{\partial\theta}\ln\left(f(x_1,\ldots,x_n;\theta)\right) = \frac{n}{\theta+1} + \sum_{i=1}^n \ln(x_i) = 0$$
$$\Rightarrow \quad \hat{\theta} = -1 - \frac{n}{\sum_{i=1}^n \ln(x_i)}$$