

STA 255 Tutorial 9

David Veitch

University of Toronto

`daveveitch.github.io`

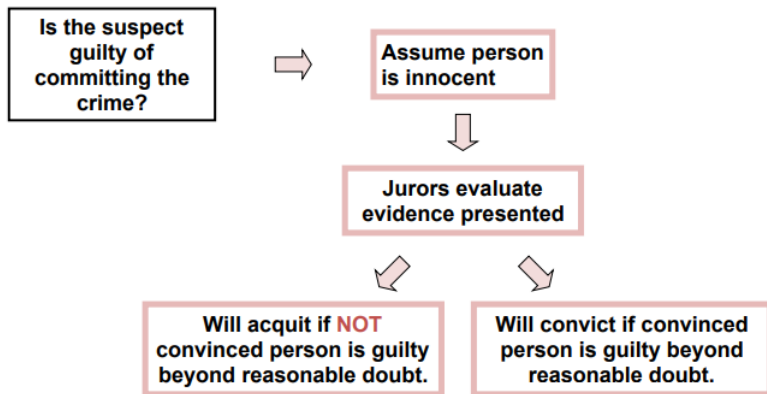
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Agenda

- 1 Kahoot
- 2 Hypothesis Testing
- 3 Confidence Intervals (Exercise 8.12 Devore & Berk)
- 4 Hypothesis Testing (Exercise 9.38 Devore & Berk)



Logic Behind Hypothesis Tests



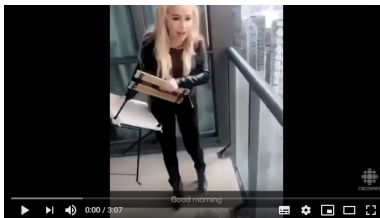
From Lecture 10 Slides

Hypothesis Testing

1. Assume Suspect Innocent



2. Evaluate Evidence



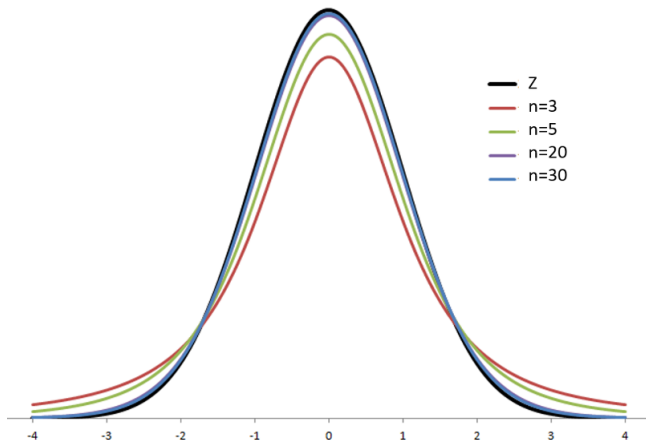
3. Decide guilty or not guilty.

Confidence Intervals (Exercise 8.12 Devore & Berk)

A random sample of 110 lightning flashes in a region resulted in a sample average radar echo duration of .81s and a sample standard deviation of .34s (*Lightning Strikes to an Airplane in a Thunderstorm, J. Aircraft, 1984: 607611*). Calculate a 99% (two-sided) confidence interval for the true average echo duration μ , and interpret the resulting interval.



Here μ and σ^2 are unknown, so we must use a t-distribution. Or do we?



Where $z_{.005} = 2.58$

$$\begin{aligned}\bar{X} \pm z_{\frac{\alpha}{2}} \times \frac{s}{\sqrt{n}} \\ .81 \pm 2.58 \times \frac{.34}{\sqrt{110}} \\ .81 \pm .084\end{aligned}$$

Therefore the 99% confidence interval we get for μ is (0.726,0.894).

What would happen if n was larger or smaller? Why does this intuitively make sense?

Hypothesis Testing (Exercise 9.38 Devore & Berk)

A random sample of 150 recent donations at a blood bank reveals that 82 were type A blood. Does this suggest that the actual percentage of type A donations differs from 40%, the percentage of the population having type A blood? Carry out a test of the appropriate hypotheses using a significance level of .01. Would your conclusion have been different if a significance level of .05 had been used?



Recall the CLT which says for a sample X_1, \dots, X_n from a distribution with mean μ variance σ^2 , and $n \rightarrow \infty$ we have:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \underset{\text{approx}}{\sim} N(0, 1)$$

Also recall a single Bernoulli random variable X_i has variance $p(1 - p)$.

For a two-sided test using critical values $z_{.005} = 2.58$, and $z_{.025} = 1.96$ we get:

$$z = \frac{\hat{p} - p_0}{\sigma/\sqrt{n}} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{82/150 - .40}{.40(.60)/150} = 3.667$$

Therefore we can reject the null hypothesis at both the 0.01 and 0.05 confidence level.